Homework 1

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EE 551 Spring 2016

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Chapter 2

Exercise 2

A three-blade wind turbine captures 1 MW from wind moving horizontally with respect to the plane. If the upwind speed is 15 m/s and the coefficient of performance is 10%, compute the length of the blade.

Known values:

$$\begin{aligned} w_u &= 15 \text{ m/s} \\ P_{turbine} &= 1 \text{ MW} \\ C_p &= 0.1 \\ \delta &= 1 \text{ kg/m}^3 \\ \text{blades} &= 3 \end{aligned}$$

Power extracted by a single blade:

$$P_b = \frac{1}{2}\delta A_b w_u^3 C_p$$

Since we have three blades and the total power from those three blades:

$$\begin{aligned} 3 \cdot P_b &= \frac{3}{2} A_b w_u^3 C_p \\ 1000000 &= \frac{3}{2}(1)(A_b)(15^3)(0.1) \\ 1000000 &= 506.25 A_b \\ \therefore A_b &= 1975.3 \text{ m}^2 \end{aligned}$$

Solving for the radius of the swept area:

$$\begin{aligned} A_b &= \pi r^2 \\ 1975.3 &= \pi r^2 \\ \therefore r &= 25.07 \text{ m} \end{aligned}$$

The length of a single blade is 17.7 meters.

Exercise 6

A wind turbine with 10 m blade length has upwind speed of 20 m/s and downwind speed of 10 m/s. Compute the power that is captured by the blade.

Known values:

$$\begin{aligned} \text{radius} &= 10 \text{ m} \\ w_u &= 20 \text{ m/s} \\ w_d &= 10 \text{ m/s} \\ \delta &= 1 \text{ kg/m}^3 \end{aligned}$$

To find the blade power while knowing upwind and downwind speeds, the following can be used:

$$P_b = \frac{1}{2} f (w_u^2 - w_d^2)$$

where flow

$$f = \delta A_b \left(\frac{w_u + w_d}{2} \right)$$

Solving for area:

$$\begin{aligned} A_b &= \pi r^2 = \pi (10)^2 \\ &= 314.15 \text{ m}^2 \end{aligned}$$

Solving for blade power:

$$\begin{aligned} P_b &= \frac{1}{2} \left[\delta A_b \left(\frac{w_u + w_d}{2} \right) \right] (w_u^2 - w_d^2) \\ P_b &= \frac{1}{2} \left[(1)(314.15) \left(\frac{20 + 10}{2} \right) \right] (20^2 - 10^2) \\ &= 706.8 \text{ kW} \end{aligned}$$

The power captured by the blade is 706.8 kW.

Exercise 8

The true wind speed is 15 m/s at an angle of 20^o with respect to the horizontal plane. A wind turbine blade with a center of gravity of 20 m from the center of the hub is rotating at 20 r/min. Compute the relative wind speed.

Known values:

$$w = 15\angle 20^o \text{ m/s}$$
$$r_c = 20 \text{ m}$$
$$f_b = 20 \text{ rev/min}$$

Converting the frequency of the blade to radians per second, the angular velocity ω_b can be calculated:

$$\begin{aligned}\omega_b &= 2\pi f_b \\ &= 20\frac{[\text{rev}]}{[\text{min}]} \cdot \frac{1[\text{min}]}{60[\text{sec}]} \cdot \frac{2\pi[\text{rad}]}{[\text{rev}]} \\ &= \frac{2\pi}{3} \text{ rad/s}\end{aligned}$$

Knowing the angular velocity, we can calculate the velocity at the given center of gravity. Note the angle of V_b is 90^o.

$$\begin{aligned}V_b = r_c\omega_b &= 20 \cdot \frac{2\pi}{3} \\ &= 41.8879\angle 90^o \text{ m/s}\end{aligned}$$

To calculate the relative wind speed, we need to know the true wind speed and the headwind speed. Since $H = -V_b$:

$$\begin{aligned}H_b &= -V_b \\ &= 41.89\angle -90^o \\ w_r &= H + w = (41.89\angle -90^o) + (15\angle 20^o) \\ &= 39.36\angle -69.02^o \text{ m/s}\end{aligned}$$

The relative wind speed is $39.36\angle -69.02^o$ m/s.

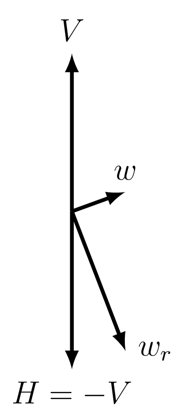

Figure 1: Diagram showing how the vectors line up.

Exercise 9

The reatlive wind speed is 15 m/s at an angle of -20^o with respect to the horizontal plane. If the pitch angle of the wind turbine is 30^o, compute the angle of attack.

Known values:

$$w_r = 15\angle - 20^o \text{ m/s}$$
$$\beta = 30^o$$

We know that the angle of attack α is between the relative wind and the camber of the blade. We know that the pitch β is between the camber and the blade direction of movement and that the direction of movement has an angle of 90^o.

$$\begin{aligned} 90 &= \theta_{w_r} + \alpha + \beta \\ \alpha &= 90 - \theta_{w_r} - \beta \\ &= 90 - 20 - 30 \\ \therefore \alpha &= 40^o \end{aligned} \tag{0.1}$$

The angle of attack is 40^o.

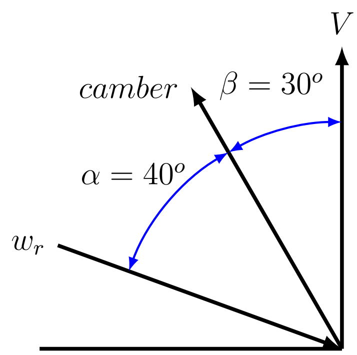

Figure 2: Diagram showing the angles.

Exercise 11

A wind turbine with 10 m blade length has upwind speed of 20 m/s and downwind speed of 10 m/s. Compute the power that can be captured by the blade.

Known values:

$$\begin{aligned} \text{radius} &= 10 \text{ m} \\ w_u &= 20 \text{ m/s} \\ w_d &= 10 \text{ m/s} \\ \delta &= 1 \text{ kg/m}^3 \end{aligned}$$

This is similar to problem 2.6:

$$P_b = \frac{1}{2} f (w_u^2 - w_d^2)$$

where

$$f = \delta A_b \left(\frac{w_u + w_d}{2} \right)$$

Solving:

$$\begin{aligned} A_b &= \pi r^2 = \pi (10)^2 = 100\pi \\ f &= (1)(100\pi) \left(\frac{20 + 10}{2} \right) \\ &= 1500\pi \\ P_b &= \frac{1}{2} (1500\pi)(20^2 - 10^2) \\ &= 225000\pi \\ &\approx 706.8 \text{ kW} \end{aligned}$$

The blade can capture 706.8 kW of power.

Exercise 12

The coefficient of performance of a wind turbine is 20% at a given pitch angle when the upwind speed is 10 m/s. The length of the blade is 50 m. Compute the flow rate of the air mass and power captured by the blades.

Known values:

$$\begin{aligned} C_p &= 0.2 \\ w_u &= 10 \text{ m/s} \\ \text{radius} &= 50 \text{ m} \\ \delta &= 1 \text{ kg/m}^3 \end{aligned}$$

Solving for power generated by blade:

$$\begin{aligned} A_b &= \pi r^2 = \pi 50^2 = 2500\pi \text{ m}^2 \\ P_b &= \frac{1}{2}\delta A_b w_u^3 C_p \\ &= \frac{1}{2}(1)(2500\pi)(10)^3(0.2) \\ &= 786.398 \text{ kW} \end{aligned}$$

From here, we need to solve for flow rate f which can be done in the following steps:

1. Use coefficient of performance equation to solve for downwind speed w_d
2. Use average wind speed to calculate w_b
3. Calculate flow rate f using blade area A_b and blade wind speed w_b

$$\begin{aligned} C_p &= \frac{1}{2}(1+\gamma)(1-\gamma^2),\ \gamma = \frac{w_d}{w_u} \\ \therefore C_p &= \frac{1}{2}\left(1+\frac{w_d}{w_u}\right)\left(1-\left(\frac{w_d}{w_u}\right)^2\right) \\ 0.2 &= \frac{1}{2}\left(1+\frac{w_d}{10}\right)\left(1-\left(\frac{w_d}{10}\right)^2\right) \\ w_d &= \{8.87, -4.80, -14.07\} \\ \therefore w_d &= 8.87 \text{ m/s} \\ f &= A_b w_b = A_b\left(\frac{w_u+w_d}{2}\right) \\ &= (2500\pi)\left(\frac{10+8.87}{2}\right) \\ &= 74102 \text{ kg/s} \end{aligned}$$

The flow rate is 74102 kg/s and the power captured by the blades is 786.398 kW.

Exercise 15

A wind developer acquires a 10 x 10 km land to install wind turbines of 50 m blade length. To achieve a separation of 8, how many wind turbines can be installed at the site?

Known values:

$$\text{radius} = 50 \text{ m}$$
$$S = 8$$

Solve for distance D:

$$S = \frac{D}{2r}$$
$$\therefore D = 2rS = 2 \cdot 50 \cdot 8$$
$$= 800 \text{ m}$$

Using the area equation to solve for the number of wind turbines in a single row:

$$A_{\text{land}} = [(X-1)D + 2r]^2$$
$$10000^2 = [(X-1)800 + 100]^2$$
$$\therefore X = 1 + int\left(\frac{9900}{800}\right)$$
$$= 13 \text{ turbines}$$

With 13 wind turbines in a single row, that is $13^2 = 169$ total wind turbines.

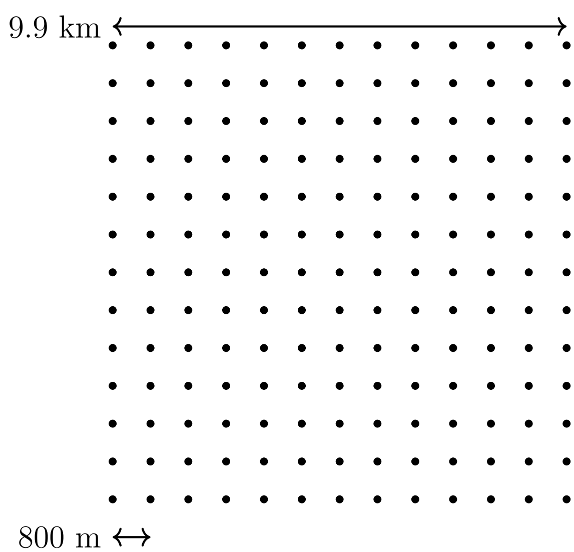

Figure 3: The field of 169 wind turbines (13x13).

Exercise 16

For the wind farm in the previous example (2.15), compute the power production per land area. Assume that the wind power density at the hub is 400 W/m², the coefficient of performance is 0.3, and the overall efficiency of the turbine-generator system is 85%.

Known values:

$$\begin{aligned} \rho &= 400 \text{ W/m}^2 \\ C_p &= 0.3 \\ \eta_{\text{turbine}} &= 0.85 \end{aligned}$$

From 2.15, known values:

$$\begin{aligned} \text{radius} &= 50 \text{ m} \\ n &= 169 \text{ wind turbines} \\ A_{\text{land}} &= 9.9\text{km x } 9.9\text{km} = 98.01 \text{ km}^2 \end{aligned}$$

First calculate the power production per wind turbine:

$$\begin{aligned} P_w &= \rho A_b = 400 \cdot 2500\pi \\ &= 3.14 \text{ MW} \\ P_{\text{turbine}} &= P_b \eta_{\text{turbine}} = P_w C_p \eta_{\text{turbine}} \\ &= 3.14 \cdot 10^6 \cdot 0.3 \cdot 0.85 \\ &= 801.106 \text{ kW} \end{aligned}$$

Total power production for 169 wind turbines:

$$\begin{aligned} P_{\text{total}} &= n \cdot P_{\text{turbine}} = 169 \cdot 801.106 \\ &= 135.387 \text{ MW} \end{aligned}$$

At this point we should include an array efficiency but we were not given one. Example 2.10 in the book uses the same size array and provides an array efficiency of 74% so I will use that number:

$$\begin{aligned} P_{\text{farm}} &= P_{\text{total}} \cdot \eta_{\text{array}} \\ &= 135.387 \cdot 0.74 \\ &= 100.19 \text{ MW} \end{aligned}$$

Lastly, the power production per land area:

$$\begin{aligned} \frac{P_{\text{farm}}}{A_{\text{land}}} &= \frac{100.19 \text{ MW}}{98.01 \text{ km}^2} \\ &\approx 1.02 \text{ MW/km}^2 \end{aligned}$$

The power production per land area is 1.02 MW/km^2.