

# Homework 6

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## Chapter 10

### Exercise 24

An eight-pole DFIG is spinning at 990 rpm. Its slip power is 10 kW. Ignore losses and compute the power delivered to the grid.

Known values:

$$\begin{aligned}n &= 990 \text{ rpm} \\p &= 8 \text{ poles} \\f &= 60 \text{ Hz} \\P_{\text{slip}} &= -10 \text{ kW}\end{aligned}$$

1. Calculate the slip:

$$\begin{aligned}n_s &= 120 \cdot \frac{f}{p} = 120 \cdot \frac{60}{8} = 900 \text{ rpm} \\s &= \frac{n_s - n}{n_s} = \frac{900 - 990}{900} = -0.1\end{aligned}$$

2. Calculate the gap power  $P_g$ :

$$\begin{aligned}P_{\text{slip}} &= sP_g \\ \therefore P_g &= \frac{P_{\text{slip}}}{s} = \frac{-10000}{-0.1} \\ \therefore P_g &= 100 \text{ kW}\end{aligned}$$

3. Calculate the supplied power  $P_s$ :

$$\begin{aligned}P_s &= P_g + P_{\text{cu1}} \\ \text{ignore losses} &\implies P_{\text{cu1}} = 0 \\ \therefore P_s &= 100000 + 0 \\ \therefore P_s &= 100 \text{ kW}\end{aligned}$$

**So the generator supplied +100 kW to the grid.**

Note the slip power is negative because of convention. Super-synchronous means negative injected power while sub-synchronous means positive injected power in this case only because we are ignoring copper losses. It is conceivable that the  $P_{\text{cu2}}$  losses in the rotor could exceed the generated power which would make the rotor windings a net consumer instead of producer. Since we are ignoring  $P_{\text{cu2}}$ , the 10 kW power in the rotor windings created by injecting voltage there is produced.

*If we include the power generated by the rotor, then there would be an additional 10 kW of power going to the grid from the rotor.*

### Exercise 29

A 690 V, six-pole DFIG is spinning at 1300 rpm. The parameters of the machine are  $r'_2 = 0.01 \Omega$ ;  $x'_2 = 0.1 \Omega$ ;  $x'_m = 2 \Omega$ . If  $2.0 \text{ V} \angle 180^\circ$  (referred to the stator) is injected into the rotor circuit, compute the developed power and the power delivered to the grid.

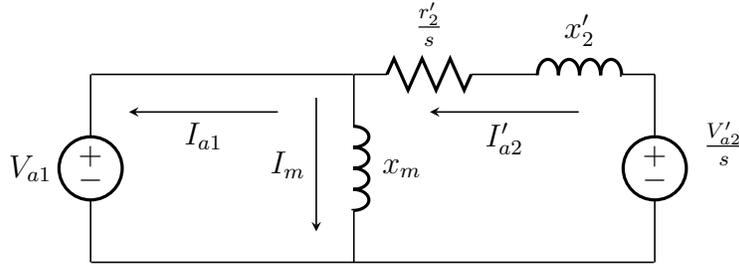
Known values:

$$\begin{aligned}\bar{V}_{a1} &= \frac{690}{\sqrt{3}} \angle 0^\circ \text{ V} \\ r_1 &\implies 0 \\ x_1 &\implies 0 \\ r'_2 &= 0.01 \Omega \\ x'_2 &= 0.1 \Omega \\ x_m &= 2 \Omega \\ \bar{V}_{a2} &= 2.0 \angle 180^\circ \text{ V} \\ n &= 1300 \text{ rpm} \\ f &= 60 \text{ Hz}\end{aligned}$$

1. Calculate the slip:

$$\begin{aligned}n_s &= 120 \cdot \frac{f}{p} = 120 \cdot \frac{60}{6} = 1200 \text{ rpm} \\ s &= \frac{n_s - n}{n_s} = \frac{1200 - 1300}{1200} = -0.0833\end{aligned}$$

2. With the stator winding impedance ignored, we get a situation like that shown in Figure 1.



**Figure 1:** The generator circuit representation showing the lack of the stator impedance.

This allows us to solve for the rotor current  $I'_{a2}$  and the magnetizing current  $I_m$ .

$$\begin{aligned}\bar{I}'_{a2} &= \frac{\bar{V}'_{a2} - \bar{V}_{a1}}{\frac{r'_2}{s} + jx'_2} = \frac{\frac{2.0 \angle 180^\circ}{-0.0833} - \frac{690}{\sqrt{3}} \angle 0^\circ}{\frac{0.01}{-0.0833} + j0.1} \\ &= 1840.99 + j1533.55 \text{ A} \\ &= 2396.04 \angle 39.79^\circ \text{ A} \\ \bar{I}_m &= \frac{\bar{V}_{a1}}{jx_m} = \frac{\frac{690}{\sqrt{3}} \angle 0^\circ}{j2.0} \\ &= 0 - j199.19 \text{ A} \\ &= 199.19 \angle -90^\circ \text{ A}\end{aligned}$$

3. Combining the magnetizing current  $I_m$  and the rotor current  $I'_{a2}$ , we calculate the stator current  $I_{a1}$ :

$$\begin{aligned} I_{a1} &= I'_{a2} - I_m \\ &= 1841.172 + j1733.496 \text{ A} \\ &= 2528.819 \angle 43.27^\circ \text{ A} \end{aligned}$$

4. Calculate the power supplied to the grid:

$$\begin{aligned} S_s &= 3\bar{V}_{a1}\bar{I}_{a1}^* \\ &= 3 \cdot \frac{690}{\sqrt{3}} \cdot (2528.819 \angle 43.27^\circ) \\ &= 2200412.649 - j2071727.207 \text{ VA} \\ P_s &= \text{Real}(S_s) = 2.200 \text{ MW} \end{aligned}$$

5. Calculate the Air Gap Power  $P_g$ :

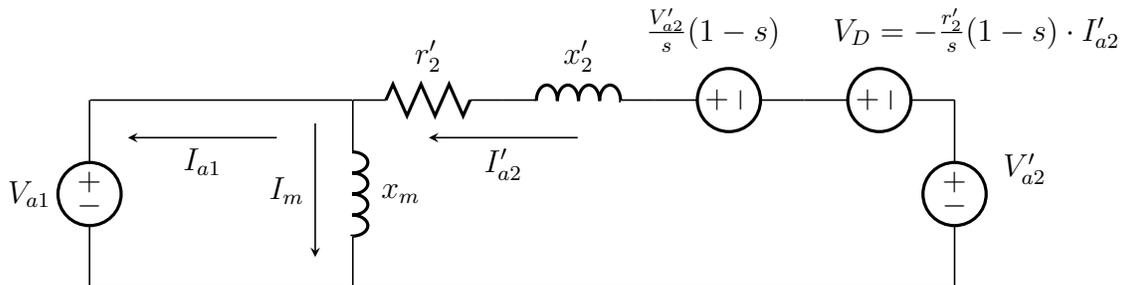
$$\begin{aligned} P_g &= P_{cu1} + P_s \\ \text{ignore stator losses} &\implies P_{cu1} = 0 \\ \therefore P_g &= P_s \end{aligned}$$

6. Calculate the developed power  $P_d$ :

$$\begin{aligned} P_d &= P_g(1 - s) \\ &= P_s(1 - s) \\ &= 2200248.09(1 - (-0.0833)) \\ \therefore P_d &= \mathbf{2.384 \text{ MW}} \end{aligned}$$

### Alternative Solution

As it happens, there appears to be another way to solve for the developed power using equations provided in the book though the result is the same. The alternative solution uses the circuit in Figure 2. In this case, the developed power voltages are split up from the rest of the circuit to make  $P = IV$  calculations more obvious.



**Figure 2:** The generator circuit from Figure 1 with the developed power  $P_d$  pieces separated from the constant values for  $V'_{a2}$  and  $x'_2$ .

1. Following Figure 2, the solution begins with solving for  $r_d$ :

$$r_d = \frac{r'_2}{s}(1 - s) = -0.13$$

2. Solve for  $V_D$ :

$$\begin{aligned} V_D &= -\frac{r'_2}{s}(1 - s) \cdot I'_{a2} \\ &= -\frac{0.01}{-0.0833}(1 - -0.833) \cdot (2396.04) \\ &= 311.567 \text{ V} \end{aligned}$$

3. Calculate the developed power portion through  $V_D$ :

$$\begin{aligned} P_{r_d} &= 3 \cdot V_D \cdot I'_{a2} \\ &= 3 \cdot (311.567) \cdot (2396.04) \\ &= 2240168.936 \text{ W} = 2.240 \text{ MW} \end{aligned}$$

4. Calculate the developed power portion through  $\frac{V'_{a2}}{s}(1 - s)$ :

$$\begin{aligned} P_{\frac{V'_{a2}}{s}} &= 3 \cdot \frac{V'_{a2}}{s}(1 - s) \cdot I'_{a2} \\ &= 3 \cdot \frac{2}{-0.0833}(1 - -0.0833) \cdot (2396.04) \\ &= -186940.192 \text{ W} \end{aligned}$$

5. Sum the total developed power:

$$\begin{aligned} P_d &= P_{r_d} + P_{\frac{V'_{a2}}{s}} \\ &= (2240168.936) - (186940.192) \\ &= 2383780.369 \text{ W} \\ \therefore P_d &= \mathbf{2.384 \text{ MW}} \end{aligned}$$

### Exercise 31

A 690 V, 600 rpm DFIG system has its Grid Side Converter (GSC) connected to the grid through a transformer whose inductive reactance is  $2 \Omega$ . Compute the output voltage of the GSC (as seen from the grid side) that would deliver 60 kVAr to the farm collection bus (FCB).

Known values:

$$\begin{aligned}\bar{V}_{a1} &= \frac{690}{\sqrt{3}} \angle 0^\circ \text{ V} \\ n &= 600 \text{ rpm} \\ x_{\text{fcb}} &= 2 \Omega\end{aligned}$$

1. Calculate the current going from the GSC to FCB:

$$\bar{I} = \frac{\bar{V}'_{\text{out}} - \bar{V}_{\text{fcb}}}{\bar{x}}$$

2. Taking  $\bar{V}_{\text{fcb}}$  as a reference:

$$\bar{I} = \frac{V'_{\text{out}}}{x} \sin(\delta) + j \frac{V_{\text{fcb}} - V'_{\text{out}} \cos(\delta)}{x}$$

3. Apparent power is:

$$\begin{aligned}\bar{S}_{\text{fcb}} &= 3 \cdot \bar{V}_{\text{fcb}} \cdot \bar{I}^* \\ &= P_r + jQ_{\text{fcb}}\end{aligned}$$

4. Subbing the current equation into the apparent power for  $\bar{I}$ :

$$\bar{S}_{\text{fcb}} = P_r + jQ_{\text{fcb}} = 3 \cdot V_{\text{fcb}} \left[ \left( \frac{V'_{\text{out}}}{x} \sin(\delta) \right) - \left( j \frac{V_{\text{fcb}} - V'_{\text{out}} \cos(\delta)}{x} \right) \right]$$

5. Finally, an equation for rotor power  $P_r$  and reactive power  $Q_{\text{fcb}}$  allows solving for the output voltage of the GSC:

$$\begin{aligned}P_r &= 3 \cdot V_{\text{fcb}} \left( \frac{\bar{V}'_{\text{out}}}{x} \sin(\delta) \right) \\ 0 &= 3 \cdot \frac{690}{\sqrt{3}} \left( \frac{V'_{\text{out}}}{2} \sin(\delta) \right) \\ \therefore \delta &= 0^\circ \\ Q_{\text{fcb}} &= 3 \cdot V_{\text{fcb}} \left( \frac{V'_{\text{out}} \cos(\delta) - V_{\text{fcb}}}{x} \right) \\ \therefore 60000 &= 3 \cdot \frac{690}{\sqrt{3}} \left( \frac{V'_{\text{out}} \cos(0^\circ) - \frac{690}{\sqrt{3}}}{2} \right) \\ \therefore V'_{\text{out}} &= 498.78 \text{ V}\end{aligned}$$

## Chapter 11

### Exercise 14

A type 4 system with synchronous generator is connected to an infinite bus through a transmission line. The infinite bus voltage is 15 kV and the output voltage of the generator as referred to the high-voltage side of the transformer is 14 kV. The transmission line inductive reactance is  $4 \Omega$ , and the reactance of the transformer is  $5 \Omega$ .

- a. Compute the capacity of the system.

Known values

$$\begin{aligned}V_g &= 15 \text{ kV} \\V'_{\text{out}} &= 14 \text{ kV} \\x_{\text{line}} &= 4 \Omega \\x_{\text{xfm}} &= 5 \Omega\end{aligned}$$

The capacity of the system is the maximum power produced which is the pullout power. The transformer and power line are in series so that  $x_{\text{total}} = 9 \Omega$ .

$$\begin{aligned}P_{\text{max}} &= \frac{V'_{\text{out}} V_g}{x} \\&= \frac{15000 \cdot 14000}{9} \\ \therefore P_{\text{max}} &= \mathbf{23.333 \text{ MW}}\end{aligned}$$

- b. If a  $2 \Omega$  capacitor is connected in series with the transmission line, compute the new capacity of the system.

The capacitor will add  $j2 \Omega$  of impedance to the line.

$$\begin{aligned}P_{\text{max}} &= \frac{15000 \cdot 14000}{9 - 2} = \frac{15000 \cdot 14000}{7} \\ \therefore P_{\text{max}} &= \mathbf{30.0 \text{ MW}}\end{aligned}$$

The addition of the capacitor increases the maximum real power significantly.

## Exercise 16

A type 4 system with synchronous generator is connected to an infinite bus through a transmission line. The infinite bus voltage is 15 kV, the transmission line inductive reactance is 6  $\Omega$ , and the reactance of the transformer is 4  $\Omega$ . If the real power delivered to the infinite bus is 5 MW, compute the output voltage of the full converter that would deliver 1 MVar at the infinite bus.

Known values:

$$\begin{aligned}V_g &= 15 \text{ kV} \\x_{\text{line}} &= 6 \Omega \\x_{\text{xfm}} &= 4 \Omega \\P_g &= 5 \text{ MW} \\Q_g &= 1 \text{ MVar}\end{aligned}$$

1. Calculate the voltage using the real power:

$$\begin{aligned}P_g &= \frac{V_g V'_{\text{out}}}{x} \sin(\delta) \\5000000 &= \frac{15000 \cdot V'_{\text{out}}}{10} \sin(\delta) \\\therefore V'_{\text{out}} \sin(\delta) &= 3.333 \text{ kV}\end{aligned}$$

2. Calculate the voltage using the reactive power:

$$\begin{aligned}Q_g &= \frac{V_g}{x} (V'_{\text{out}} \cos(\delta) - V_g) \\1000000 &= \frac{15000}{10} (V'_{\text{out}} \cos(\delta) - 15000) \\\therefore V'_{\text{out}} \cos(\delta) &= 15.667 \text{ kV}\end{aligned}$$

3. Calculate the power angle:

$$\begin{aligned}\frac{V'_{\text{out}} \sin(\delta)}{V'_{\text{out}} \cos(\delta)} &= \frac{3333}{15667} \\\therefore \frac{\sin(\delta)}{\cos(\delta)} &= \frac{3333}{15667} \\\therefore \tan(\delta) &= \frac{3333}{15667} \\\therefore \delta &= 12.0^\circ\end{aligned}$$

4. Solve for the output voltage:

$$\begin{aligned}V'_{\text{out}} \cos(\delta) &= 15.667 \text{ kV} \\\therefore V'_{\text{out}} &= \mathbf{16.017 \text{ kV}}\end{aligned}$$